Mixing Efficiency and Equity in Spatial Planning

Capacitated p-Median Problem with Territorial Coverage Constraint

Felipe Albuquerque[1,2],
Rosa Figueiredo[1], Cyrille Genre-Grandpierre[2]

[1]Laboratoire d'Informatique d'Avignon (LIA)

[2]UMR ESPACE, Avignon, France

June 3, 2025

Part I

Context

Optimization in Spatial Planning: Location Problems

Context

- Limited budgets, resources, and workers
- \+ Growing concentration of population
- = Spatial inequalities in accessibility
- Affects key sectors: public health, waste management, emergency services, etc.
- Central challenge: **Where and how to allocate resources smartly?**

Optimization in Spatial Planning: Location Problems

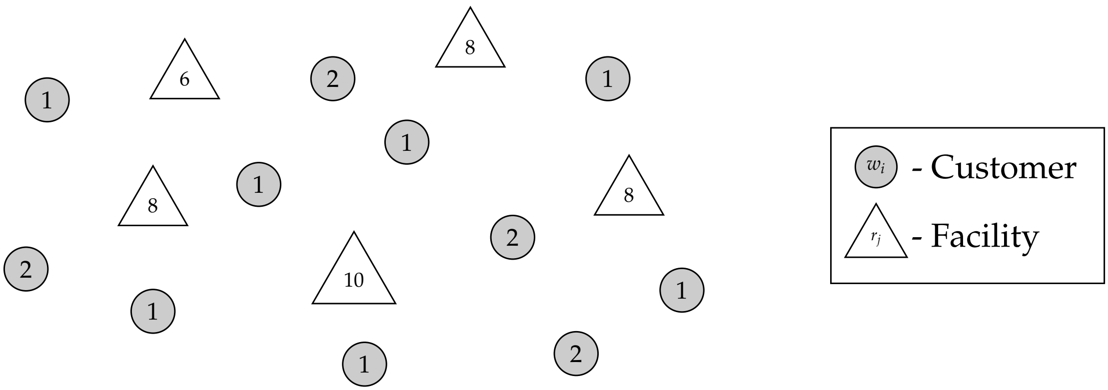
Context

- Limited budgets, resources, and workers
- \+ Growing concentration of population
- = Spatial inequalities in accessibility
- Affects key sectors: public health, waste management, emergency services, etc.
- Central challenge: **Where and how to allocate resources smartly?**
- Optimization methods serve as decision-support tools in real-world spatial planning.

What is a Location Problem?

Instance: (C, w, N, r, p)

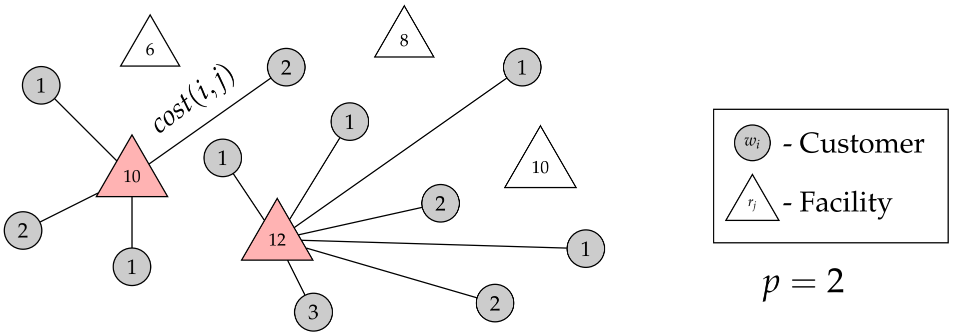
- The set C of customers
- Set N with all possible locations to install the facilities
- Number of p (facilities to install)
- w_i defining the value of demand for each customer $i \in C$
- r_j representing the value of capacity for each location $j \in N$



Objective Function (Efficiency)

- F_s represents the set of assigned customer-facility pairs (i, j).
- **Minimize** the total **allocation cost**:

$$\min \sum_{(i,j)\in F_s} \text{cost}_{ij} \Rightarrow \min \sum_{(i,j)\in F_s} \underbrace{w_i}_{\text{demand}} \cdot \underbrace{d_{ij}}_{\text{distance}}$$



$p = 2$

Classical Formulation (CpMP)

[RS70]

Variables

$y_j \in \{0,1\}$: If a facility is located in j.

$x_{ij} \in \{0,1\}$: If the customer i is allocated to facility j.

Model

$$(\mathscr{F}) \quad \min \sum_{i \in C} \sum_{j \in N} cost(i,j) x_{ij}$$

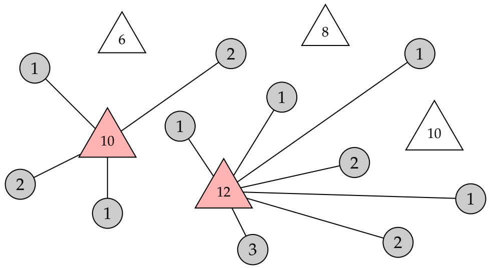
$$\text{s.t.} \quad \sum_{j \in N} x_{ij} = 1, \quad \forall i \in C,$$

$$\sum_{j \in N} y_j = p,$$

$$\sum_{i \in C} W_i x_{ij} \leq R_j y_j, \quad \forall j \in N,$$

$$x_{ij} \in \{0,1\}, \quad \forall i \in C, \forall j \in N,$$

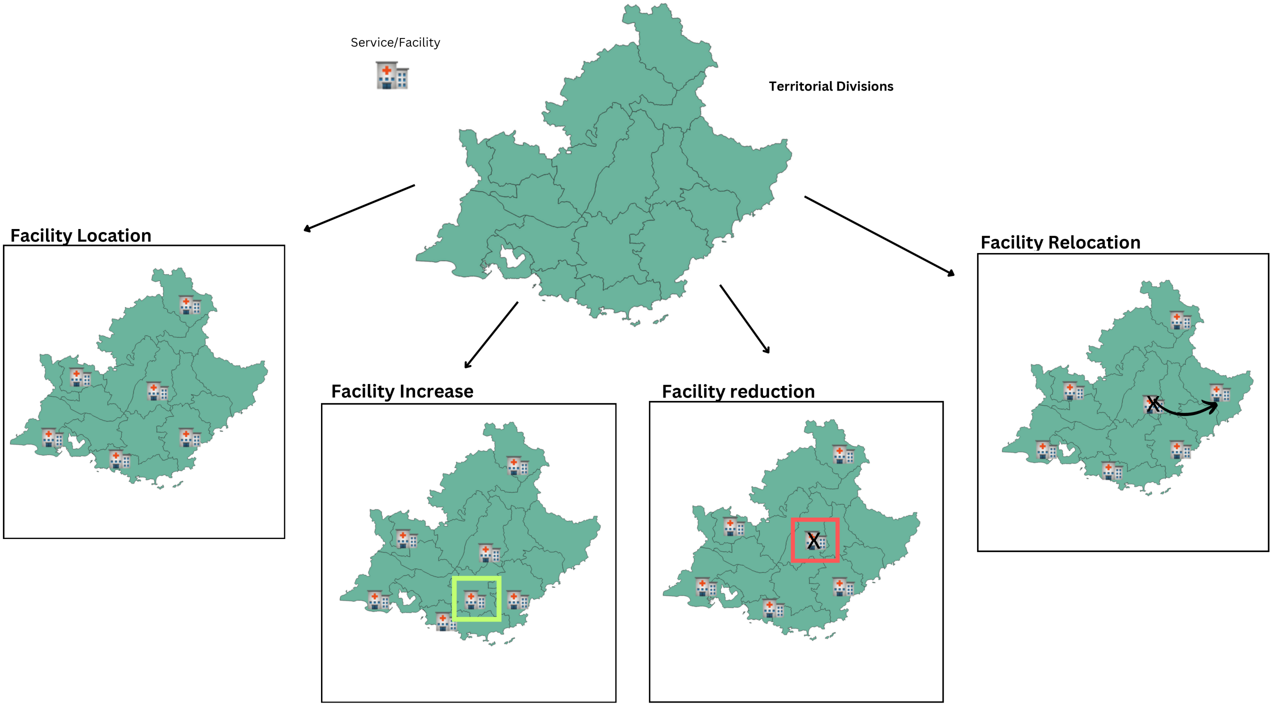
$$y_j \in \{0,1\}, \quad \forall j \in N.$$



Defining

- N: set of possible Facilities sites
- C: set of Customers
- R_j: capacity of a facility $j \in N$
- W_i: demand of a customer $i \in C$
- $cost(i,j)$: allocation cost between $i \in C$ and $j \in N$

CONSIDERING TERRITORIAL DIVISIONS IN LOCATION PROBLEMS



THE RELEVANCE OF TERRITORIAL DIVISIONS IN DECISION-MAKING

Population & Avenir 2019/5 n° 745

Article de revue

Géographie inégalitaire des services publics et aménagement du territoire

Par **Gwénaël Doré**

Pages 4 à 8

PAYS DE LA LOIRE

L'accès aux soins se dégrade dans les zones rurales

Insee Flash Pays de la Loire • n° 137 • Mars 2023

Revue d'Économie Régionale & Urbaine 2023/3 Juin

Article de revue

Services publics – Services privés : des logiques de fermeture semblables entraînant un délaissement des territoires ?

L'exemple des agences bancaires du Crédit Agricole, des collèges, postes, maternités et gendarmeries en région SUD de 2007 à 2017

Par **Quentin Godoye** et **Cyrille Genre-Grandpierre**

Pages 411 à 432

AUVERGNE-RHÔNE-ALPES

Dans le rural,
l'accès à un médecin généraliste
est difficile pour un habitant sur trois

Insee Analyses Auvergne-Rhône-Alpes • n° 187 • Décembre 2024

Revue française d'économie 2025/1 Vol. XXXIX

Article de revue

Fractures nationales : retrait des services publics et dynamiques électorales

Par **Nur Bilge**, **Étienne Farvaque** et **Jan Fidrmuc**

Pages 213 à 252

Optimization in Spatial Planning: Location Problems

Objective of the work

- Develop a model that balances the trade-offs between **efficiency** and **equity** in spatial allocation
- Design an open-source, user-friendly **location-allocation solver** to support others real-world planning decisions *(in progress)*

Combines efficiency and spatial units coverage constraint

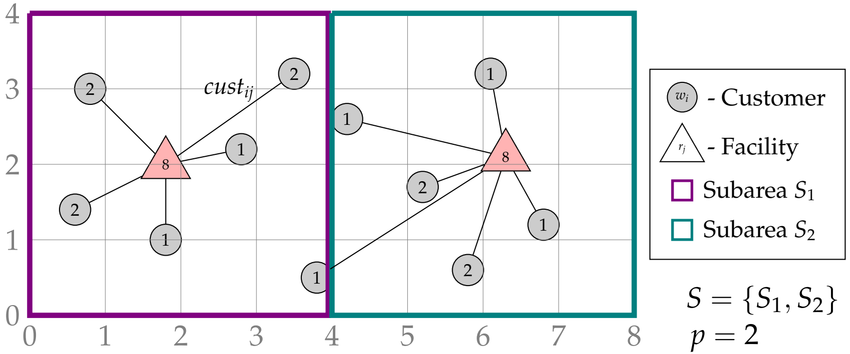
Proposed Modification

Objective Function

- **Minimize** the total **allocation cost (efficiency)**

Additional Constraint

- A set $S = \{S_1, S_2, \ldots, S_{m^s}\}$, where for k and q two distinct subareas: $S_k \cap S_q = \emptyset$
- **Cover** as many subareas as possible (**equity** dimensions).



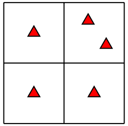
What change in the model?

Coverage Constraint

Let $N(S_k) \subseteq N$ be the subset of possible Facilities locations placed in the subarea S_k. The cover constraint depends on the value of p.

If $p \geq |S|$: Cover all subareas

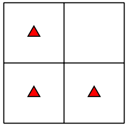
$$\sum_{j \in N(S_k)} y_j \geq 1, \quad k = \{1, 2, ..., s\}.$$



$|S| = 4$
$p = 5$

If $p \leq |S|$: Cover the most subareas

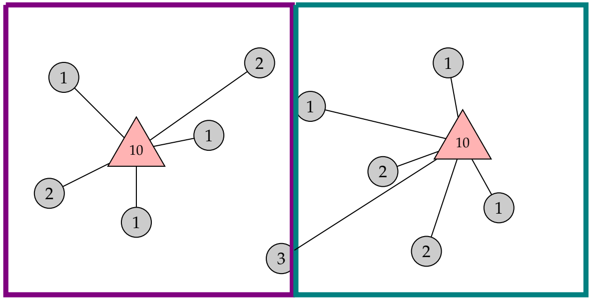
$$\sum_{j \in N(S_k)} y_j \leq 1, \quad k = \{1, 2, ..., s\}.$$



$|S| = 4$
$p = 3$

Formulation with Subareas Coverage

Adding Coverage Constraints



Constant

$\alpha \in \{0,1\}$: If the p is bigger than $|S|$.

Variables

$y_j \in \{0,1\}$: If a facility is located in j.

$x_{ij} \in \{0,1\}$: If the customer i is allocated to facility j.

Model

$$(\mathscr{F}(S)) \quad \min \sum_{i \in C} \sum_{j \in N} cost(i,j) x_{ij}$$

$$\text{s.t.} \quad \text{Same Constraints of } (\mathscr{F}),$$

$$\alpha \sum_{j \in N(S_k)} y_j \geq 1, \quad k = \{1, 2, \ldots, m^s\}$$

$$(1-\alpha) \sum_{j \in N(S_k)} y_j \leq 1, \quad k = \{1, 2, \ldots, m^s\}$$

$$x_{ij} \in \{0,1\}, \quad \forall i \in C, \forall j \in N,$$

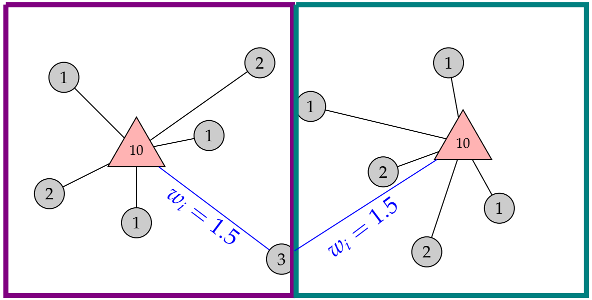
$$y_j \in \{0,1\}, \quad \forall j \in N.$$

Defining

- N: set of possible Facilities sites
- C: set of Customers
- S: set of Subareas
- R_j: capacity of a facility $j \in N$
- W_i: demand of a customer $i \in C$
- $cost(i,j)$: allocation cost between $i \in C$ and $j \in N$

Relaxed Formulation CpMP^r-SC

Coverage Constraints



Variables

$y_j \in \{0,1\}$: If a facility is located in j.

$x_{ij} \in [0,1]$: The **fraction** of demand at i allocated to facility j.

Model

$$(\mathscr{F}^r(S)) \quad \min \sum_{i \in C} \sum_{j \in N} cost(i,j) x_{ij}$$

$$\text{s.t.} \quad \text{Same Constraints of } (\mathscr{F}),$$

$$\alpha \sum_{j \in N(S_k)} y_j \geq 1, \quad k = \{1, 2, \ldots, m^s\}$$

$$(1-\alpha) \sum_{j \in N(S_k)} y_j \leq 1, \quad k = \{1, 2, \ldots, m^s\}$$

$$x_{ij} \in [0,1], \quad \forall i \in C, \forall j \in N,$$

$$y_j \in \{0,1\}, \quad \forall j \in N.$$

Defining

- N: set of possible Facilities sites
- C: set of Customers
- S: set of Subareas
- R_j: capacity of a facility $j \in N$
- W_i: demand of a customer $i \in C$
- $cost(i,j)$: allocation cost between $i \in C$ and $j \in N$

Part II

Numerical Experiements

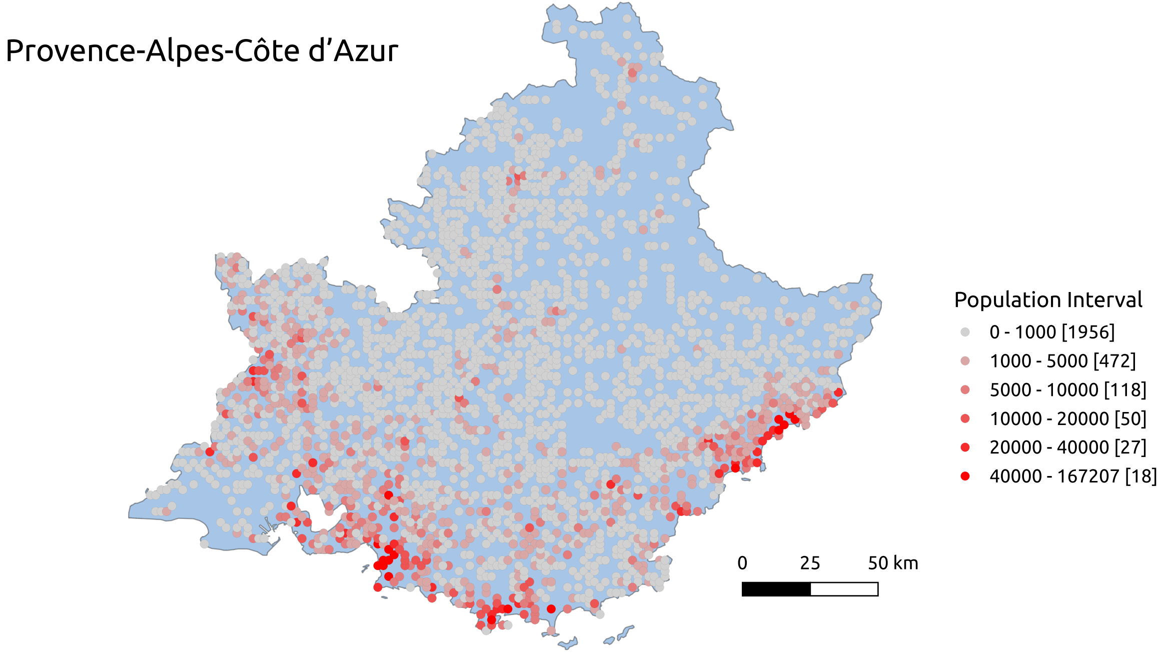
Numerical Experiments

PACA Instance

Machine Configuration

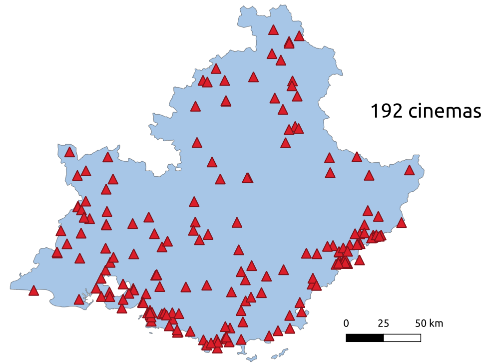
- Models: CpMP^r and CpMP^r-SC
- Language: C++
- Solver: CPLEX 22.1.1 (Concert API)
- RAM: 128GB
- OS: Ubuntu 22.04 LTS
- Time Limit: 5 hours per instance

Numerical Experiments
PACA Instance



Numerical Experiments
Parameters

Parameter	Description
Service	**Cinema**
Problems	CpMP^r and CpMP^r-SC
$N = C$	**2641** points in the region
S	EPCI (**51**), Cantons (**192**), Communes (**959**)
p	$\{134, 173, \mathbf{192}, 211, 250\}$
R_j	Created using the real 192 cinema locations
W_i	Number of inhabitants
d_{ij}	Travel time by car
cost(i, j)	$W_i \cdot d_{ij}$



Numerical Experiments

PACA Tests Details

Territorial Coverage

Evaluate how solution quality changes when **territorial coverage constraints** are added to the classical efficiency-based model

Territorial Morphology

Analyze solutions obtained by dividing the PACA region using **different spatial partitions**.

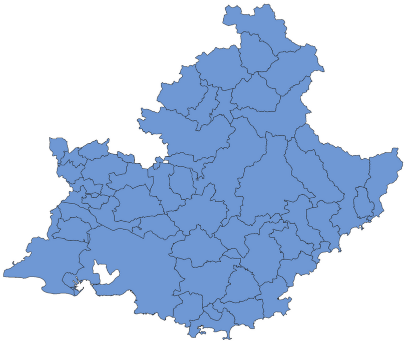
Population Distributions

Analyze the impact of various **population distributions** (no coverage constraints considered).

$$\textbf{Relative Increase in Cost } (\%) = \left(\frac{\text{Solution} - \text{Solution}_{nocover}}{\text{Solution}_{nocover}} \right) \times 100$$

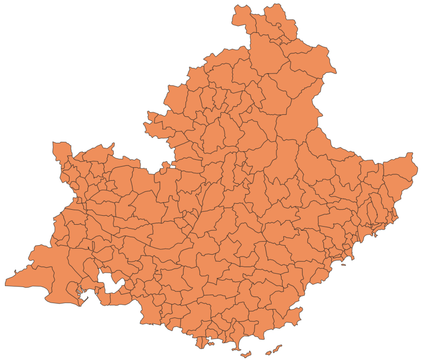
Territorial Coverage

PACA Territorial Divisions



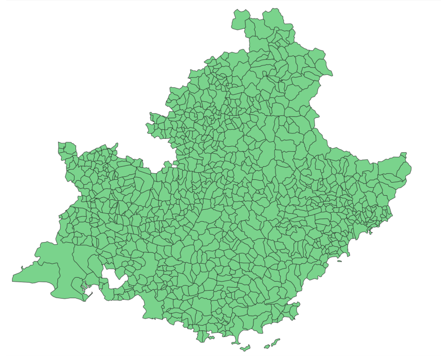
Établissements publics de coopération intercommunale (EPCI)

51



Répartition par cantons

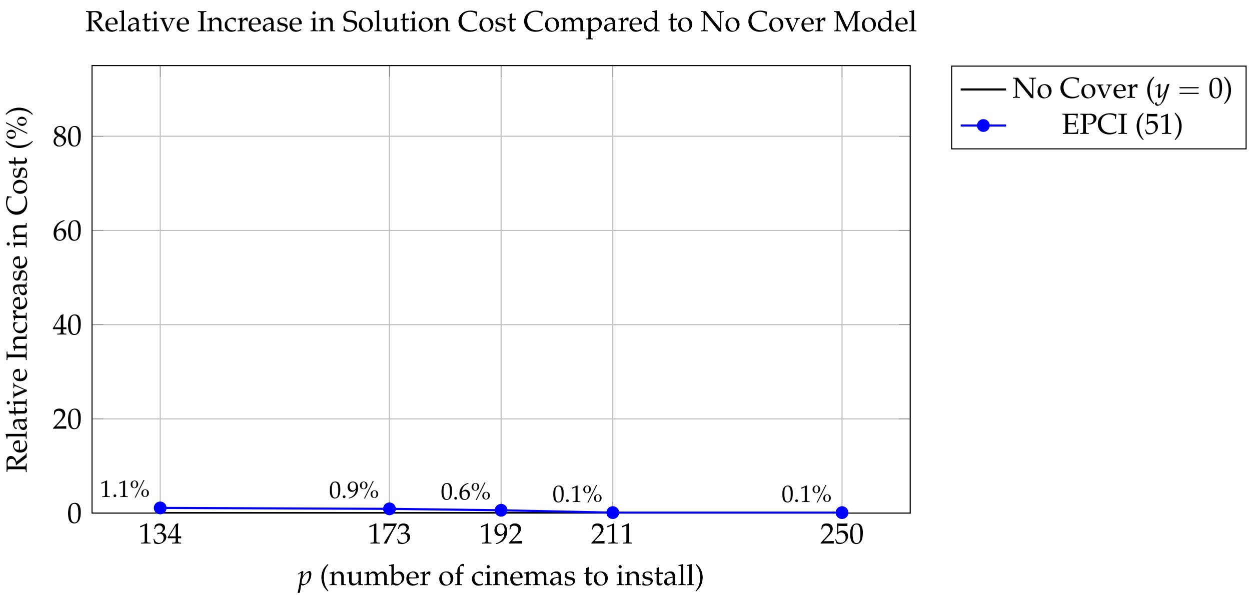
192



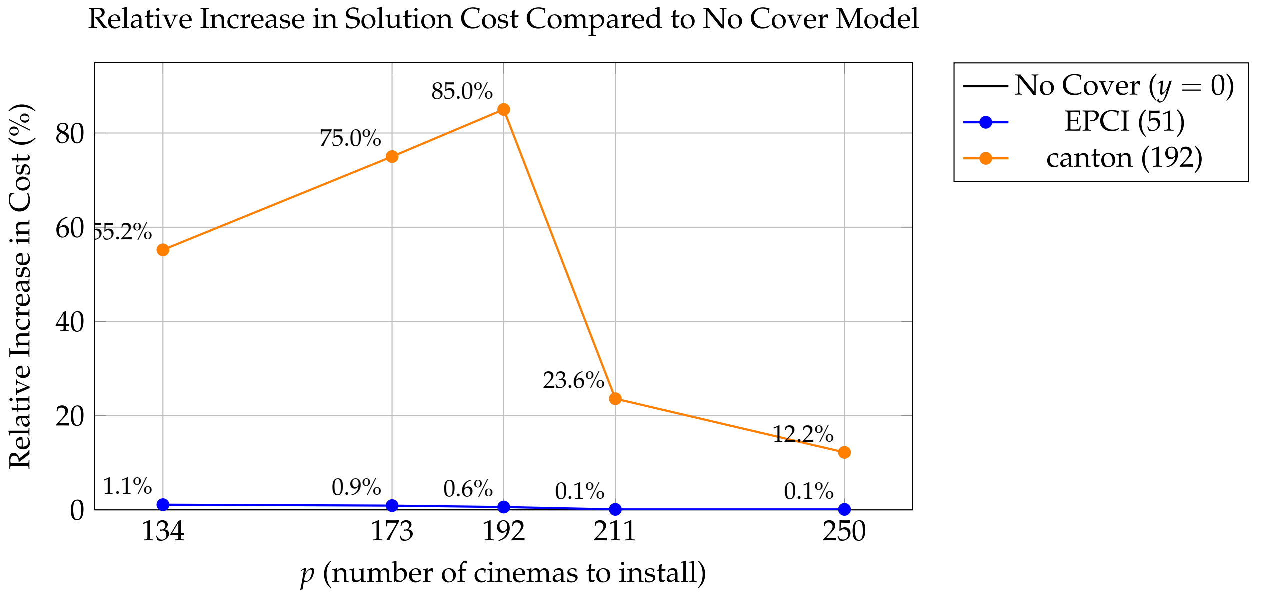
Répartition par communes

959

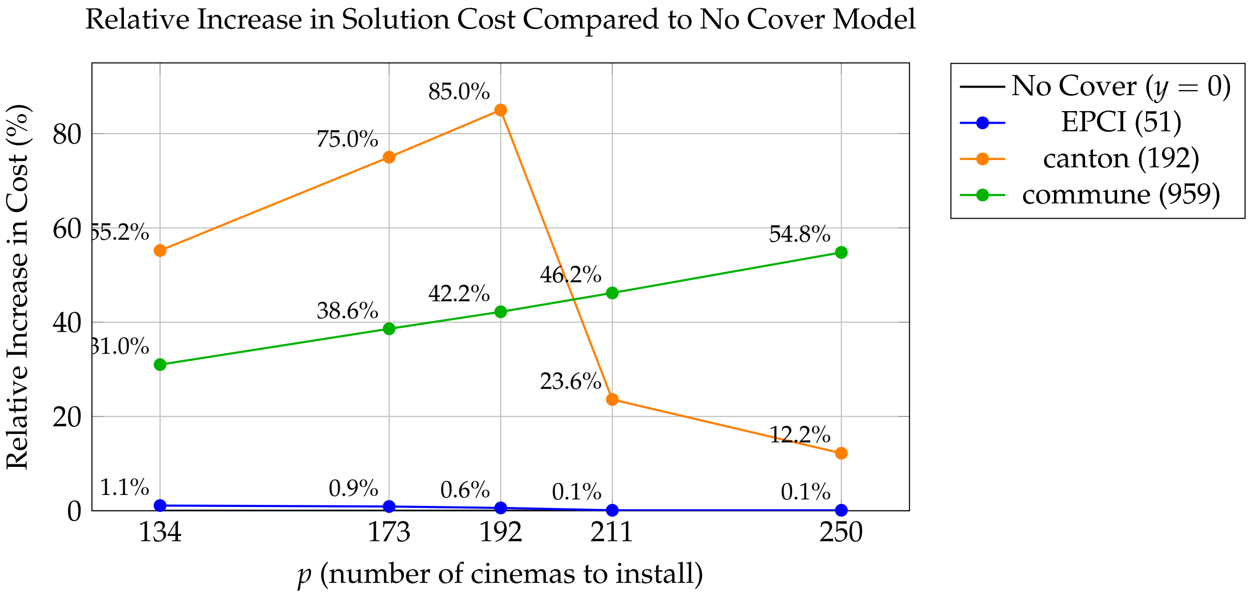
Comparison of Solutions With and Without Territorial Divisions



Comparison of Solutions With and Without Territorial Divisions



Comparison of Solutions With and Without Territorial Divisions



Coverage cost increases until p equals the number of subareas.

Analysis of the solutions when $p = 192$

Statistics

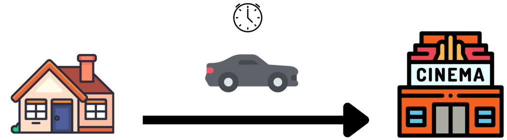


Table. Assigned demand distribution by travel time when $p = 192$ cinemas and 192 cantons

Interval (minutes)	**Real (%)**	**NoCover (%)**	**Cover Canton (%)**
0–20	74.202	96.255	80.512
20–40	17.828	3.452	18.461
40–60	6.979	0.289	1.027
60+	0.991	0.003	0.000
Avg distance (min)	22.99	17.80	14.41
Std deviation (min)	14.83	11.58	7.94
Max distance (min)	77.89	73.83	57.73

Overview of solutions characteristics

ANALYSIS OF THE SOLUTIONS WHEN $p = 192$

STATISTICS

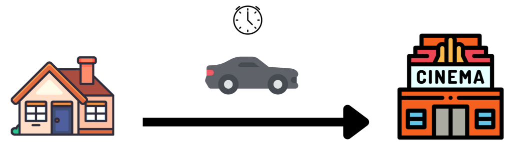


Table. Assigned demand distribution by travel time when $p = 192$ cinemas and 192 cantons

Interval (minutes)	**Real (%)**	**NoCover (%)**	**Cover Canton (%)**	
0–20	74.202	96.255	80.512	**-15.743%**
20–40	17.828	3.452	18.461	
40–60	6.979	0.289	1.027	
60+	0.991	0.003	0.000	
Avg distance (min)	22.99	17.80	14.41	
Std deviation (min)	14.83	11.58	7.94	
Max distance (min)	77.89	73.83	57.73	

Change in short travel intervals (0–20 min)

Analysis of the solutions when $p = 192$

Statistics

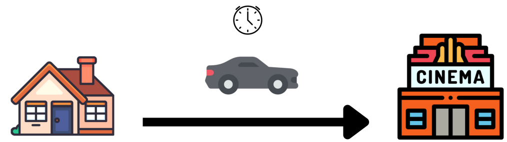


Table. Assigned demand distribution by travel time when $p = 192$ cinemas and 192 cantons

Interval (minutes)	Real (%)	NoCover (%)	Cover Canton (%)	
0–20	74.202	96.255	80.512	
20–40	17.828	3.452	18.461	
40–60	6.979	0.289	1.027	
60+	0.991	0.003	0.000	
Avg distance (min)	22.99	17.80	14.41	**-3.39**
Std deviation (min)	14.83	11.58	7.94	**-3.64**
Max distance (min)	77.89	73.83	57.73	**-16.10**

Reduction in average, std. deviation and max travel time

Analysis of the solutions when $p = 192$

Statistics

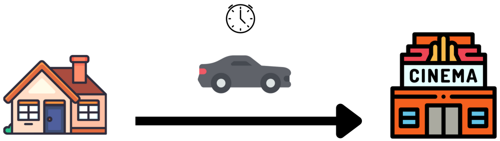
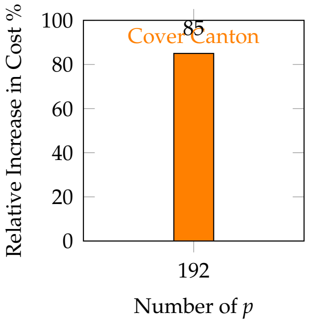


Table. Assigned demand distribution by travel time when $p = 192$ cinemas and 192 cantons

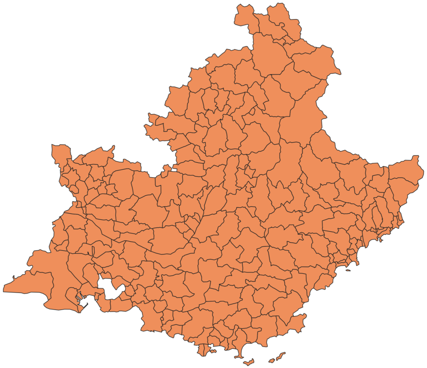
Interval (minutes)	Real (%)	NoCover (%)	Cover Canton (%)
0–20	74.202	96.255	80.512
20–40	17.828	3.452	18.461
40–60	6.979	0.289	1.027
60+	0.991	0.003	0.000
Avg distance (min)	22.99	17.80	14.41
Std deviation (min)	14.83	11.58	7.94
Max distance (min)	77.89	73.83	57.73



Territorial Morphology (PACA)

Different ways to divide the PACA region

Canton example with Grid and Voronoi divisions

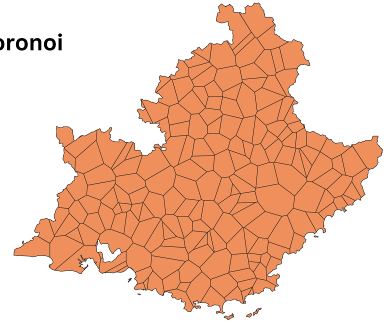


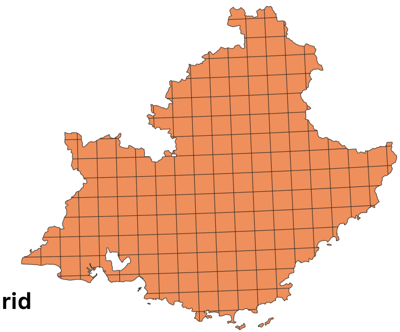
Canton real division

192

“Canton” Voronoi

192

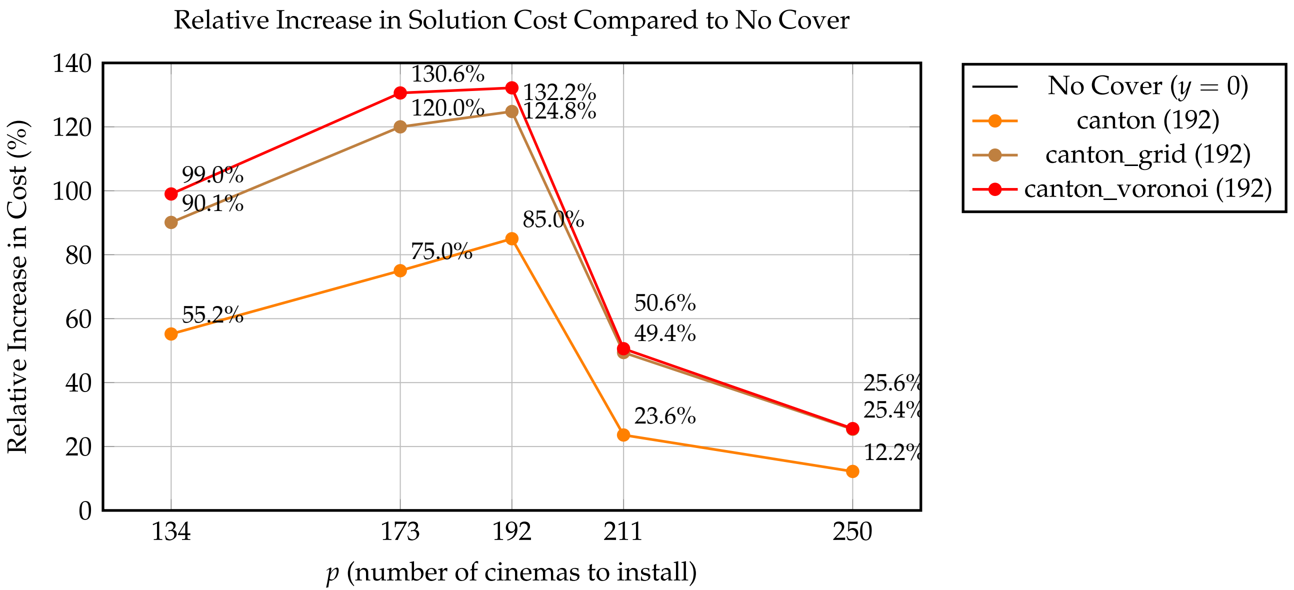




192

“Canton” Grid

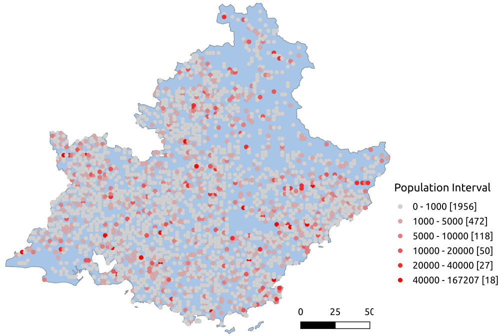
COMPARISON BETWEEN THE SOLUTIONS AND DIFFERENT TERRITORIAL MORPHOLOGY



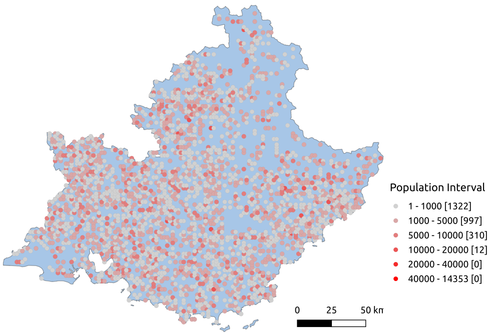
Increasing the size of regions with the highest demand impacts the cost, even under homogeneous divisions.

Population Distributions (PACA)

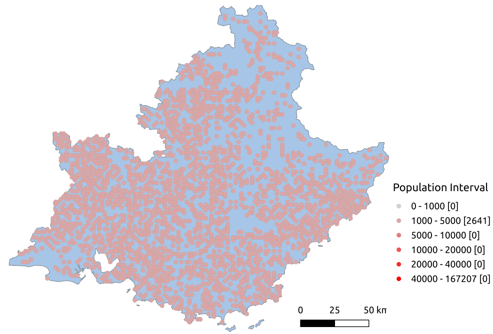
Different Population Configurations



Shuffle population

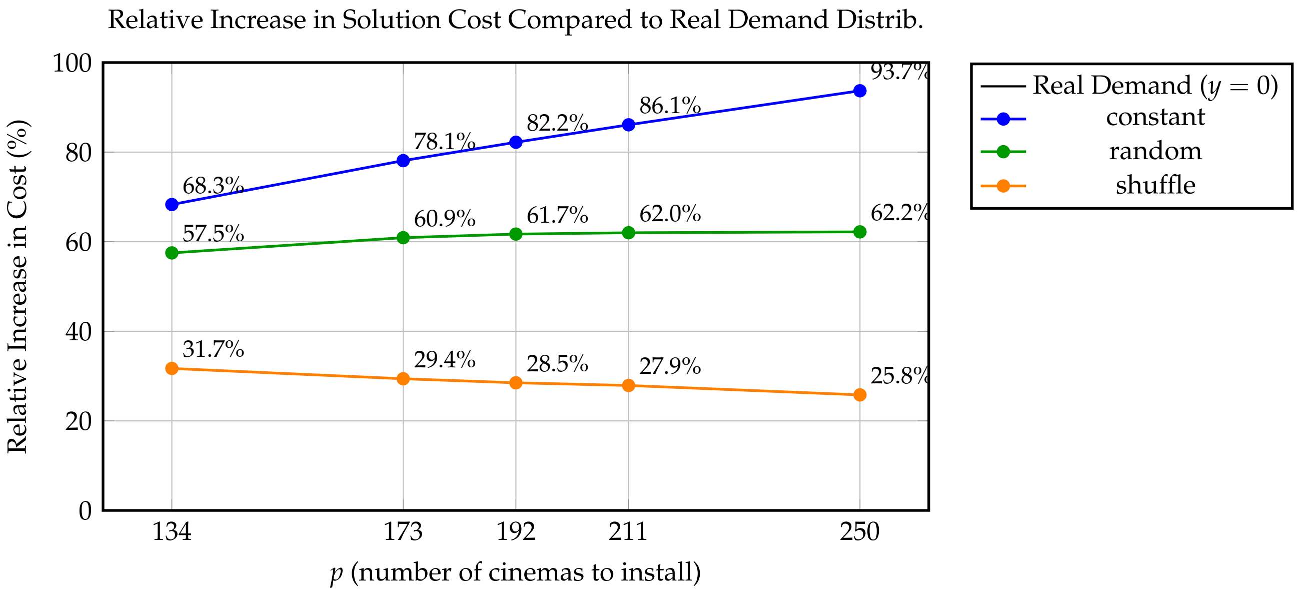


Random population



Constant population

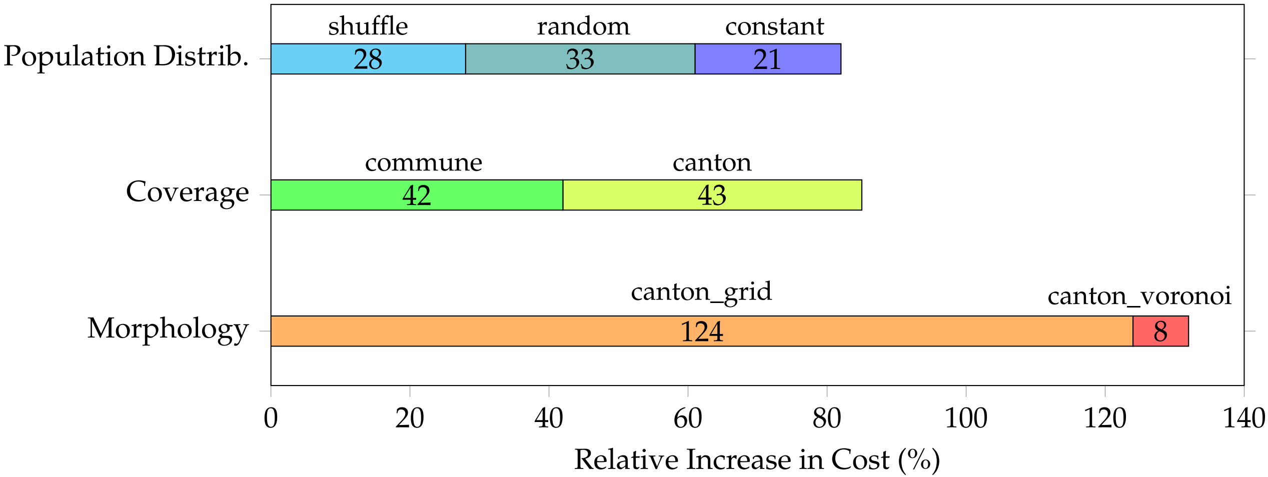
Comparison between the solutions and different population configuration



A more homogeneous or less concentrated demand distribution leads to higher installation costs.

Comparing the different Scenarios

Relative Increase in Cost in Different Scenarios for $p = 192$



When the value of p approaches the number of subareas to cover, the cost associated with territorial coverage can have a greater impact on the solution than variations in population distribution.

Part III

Conclusion and Future Goals

Conclusion and Future Work

Contributions:

- New variants: p-Median Problem + Territorial Coverage Constraints
- Adapt the existing Classical Model for the CpMP
- Matheuristic method (for large instances and fast solutions)

In progress:

- Computational Experiments of multi-scale territorial coverage constraints.
- Use lexicographic optimization with weighted subareas.
- Extend the model to handle multiple services simultaneously.

References I

[GB21] Mario Gnägi and Philipp Baumann. **“A matheuristic for large-scale capacitated clustering”.** In: *Computers & operations research* 132 (2021), p. 105304.

[LNS19] Gilbert Laporte, Stefan Nickel, and Francisco Saldanha-da-Gama. **“Introduction to location science”.** In: *Location science*. Springer, 2019, pp. 1–21.

[MT20] Wangshu Mu and Daoqin Tong. **“On solving large p-median problems”.** In: *Environment and Planning B: Urban Analytics and City Science* 47.6 (2020), pp. 981–996.

[RS70] Charles ReVelle and Ralph Swain. **“Central Facilities Location”.** In: *Geographical Analysis* 2 (Sept. 1970), pp. 30–42. DOI: `10.1111/j.1538-4632.1970.tb00142.x`.

[SAM15] Fernando Stefanello, Olinto CB de Araújo, and Felipe M Müller. **“Matheuristics for the capacitated p-median problem”.** In: *International Transactions in Operational Research* 22.1 (2015), pp. 149–167.

Thank you!